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Forecast of the Distribution of Hydrocarbon Accumulations by Size. Status and Issues

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Abstract: The most important element of the structure of hydrocarbon resources is their distribution by accumulations of different sizes. The formalism of this direction is developed in detail. Estimates of the resource structure of the largest oil and gas provinces of Russia are carried out. However, the lack of a substantiated theoretical model for the formation of the distribution of accumulations by size gives rise to a number of problems, including practical ones. This concerns the forecast of the number of small accumulations and hydrocarbon resources concentrated in them in well-studied oil and gas systems, the forecast of distribution for poorly studied systems, and the identification of the relationship between distribution parameters and the geological characteristics of the oil and gas system. The explanation of some empirically established regularities, in particular, the stable nature of the value of the distribution parameter $\lambda \sim 2$, is of independent theoretical interest. The paper presents the current state of this complex of problems and some possible directions for their solution.

Keywords: oil and gas system, structure of hydrocarbon resources, distribution by size of deposits, truncated Pareto distribution.

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Research Article

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1. Introduction

The most important element of the structure of hydrocarbon resources is their distribution by accumulations of different sizes. Research in this area emerged in the sixties and seventies of the twentieth century [*Arps et al.*, 1958; *Kaufman*, 1963]. The distribution of the sizes of the identified deposits in most cases was satisfactorily approximated by a logarithmically normal law. Analysis of changes in the sets of identified hydrocarbon accumulations with increasing exploration of the study area led to the assumption that the distribution of accumulations in nature is described by the Pareto power law with the value of the parameter λ close to 2 [*Shpilman*, 1972]:

$$\varphi(\theta) = \frac{C}{\theta^{\lambda}}, \ \theta_0 \le \theta \tag{1}$$

Here $\varphi(\theta)$ is the distribution density function, θ is the size of the accumulation, λ, θ_0 are the distribution parameters, C is the normalizing factor.

Based on data from well-studied basins of the world, A. E. Kontorovich and V. I. Demin established that the distribution of hydrocarbon accumulations by the size of reserves is described by the truncated Pareto distribution they proposed [*Kontorovich et al.*, 1977; 1979]:

$$\varphi(\theta) = C \cdot \left(\frac{1}{\theta^{\lambda}} - \frac{1}{\theta_{\max}^{\lambda}}\right), \ \theta_0 \le \theta \le \theta_{\max}, \tag{2}$$

where $\theta_0, \theta_{\max}$ are respectively the minimum and maximum sizes of the cluster taken into account, C is the normalizing factor.

The results of V. I. Shpilman, A. E. Kontorovich, V. I. Demin were confirmed by the works of N. A. Krylov, Yu. A. Arsiriy and others.

The development of this direction led to a detailed development of the formal aspects of the application of the truncated Pareto distribution and an assessment of the structure of resources of the largest oil and gas provinces of Russia [*Kontorovich et al.*, 2017; 2021].

Nevertheless, a number of questions have not found convincing answers. Among them are:

1. To what extent are the type and parameters of the distribution established empirically for the right "tail" of the distribution of the studied large oil and gas systems (OGS) adequate for the left, understudied interval of the natural set of accumulations?
2. For poorly studied OGS, the problem of substantiating the type, parameters of the approximating distribution and their connection with the characteristics of the OGS is even stronger.
3. The empirical fact of the stable gravitation of the parameter λ in (2) to the value ~ 2 cannot be explained.

All these questions cannot be resolved without developing a sufficiently clear and meaningful model of the formation of distributions of hydrocarbon accumulations by size.

2. Methods and Results

Attempts to theoretically substantiate the type of accumulations' distribution and establish the relationship between its parameters and the characteristics of the host geological objects can be found in [*Kontorovich et al.*, 1988] and etc. These works show that the power-law nature of the distribution of accumulations is probably determined by the relationship between the processes of accumulation and dissipation of hydrocarbons in the deposit. Examples of the implementation of this approach are given below.

I. It is natural to consider the size θ of an individual hydrocarbon accumulation in the OGS as a consequence of accumulation and dissipation processes. As a first approximation, the changes in θ over time can be represented as [*Burshtein*, 2004]:

$$\frac{d\theta}{\mathrm{d}t} = a - b \cdot \theta. \tag{3}$$

Here t is the time, a is the rate of hydrocarbon entry into the trap, b is the relative rate of destruction of the accumulation.

The parameters a and b can be interpreted as random variables, independent of time for long stages of OGS development. Then θ reaches a stationary value:

$$\frac{d\theta}{\mathrm{d}t} = 0, \ \theta = \frac{a}{b}.$$

For statistically independent a and b, the density of the distribution of hydrocarbon accumulation $\varphi(\theta)$ has the form:

$$(\theta) = \frac{1}{\theta^2} \int_{\Omega_a} \varphi_1(a) \cdot \varphi_2\left(\frac{a}{\theta}\right) \cdot a \cdot da. \tag{4}$$

Here Ω_a is the integration domain depending on θ; φ_1 and φ_2 are the probability densities for the distributions of hydrocarbon accumulations in the OGS by a and b.

In the particular case where the parameters a and b are distributed uniformly, the explicit form of the density $\varphi(\theta)$ for sufficiently large θ is identical to the truncated Pareto distribution (2) with the exponent $\lambda = 2$.

It is difficult to expect a uniform distribution of the parameters a and b in real OGS. In the general case, the behavior of distribution (4) on the left tail (i.e. in the area of small accumulations) differs to the direction of decreasing the proportion of accumulations compared to the truncated Pareto distribution (2). Naturally, this circumstance is significant only for values $\theta > \theta_0$. On the right tail, distribution (4) retains a power-law character, with exponents λ depending on the type of φ_1 and φ_2.

Model (3) allows us to identify the qualitative influence of the age of the OGS on the shape of the distribution density $\varphi(\theta)$ and the value of the parameter λ [*Burshtein*, 2006].

The model considered above does not take into account the spatially distributed nature of the migration process, which precedes the process of hydrocarbon accumulation and should influence the type of distribution of accumulations.

II. It is accepted that secondary migration of oil occurs in the form of movement of primary accumulations. The pore medium is defined by a random field with an exponential correlation function. The speed of movement of the primary accumulation (globules) is determined by the forces of gravitational ascent. Multiple modeling of migration is performed at different values of the parameters of porosity distribution, permeability, and formation inclination angle, taking into account random fluctuations [*Livshits*, 2017]. The sought characteristics and their fluctuations were statistically estimated from a set of such realizations. This allowed us to identify the conditions under which the lateral migration of accumulations leads to the formation of their mass distribution corresponding to the Pareto distribution.

1. The main factor determining the occurrence of a power-law distribution is the distance from the centers of generation to the traps: a power-law distribution does not occur if the traps are located too close to the centers; a large distance from the traps also reduces the probability of occurrence of a power-law distribution, since in this case the globules suffer significant migration losses. Thus, for the occurrence of a power-law distribution, the process of globule migration is necessary, but without significant migration losses.
2. The influence of the filtration-capacitive and geometric parameters of the globule migration paths on the occurrence of a power-law distribution is small, and with an increase in the dispersion of these values it weakens.
3. The latter circumstance explains the fundamental nature of the power-law distribution of hydrocarbon accumulations by mass: it has a high probability of occurrence in a wide range of changes in the parameters of migration routes, i.e. it does not depend on the individual characteristics of the basin.

III. Among the approaches that justify the emergence of distributions with a "heavy", power-law tail, a large group consists of models based on the presence of a random component ("noise") in the system.

Let us assume that the change in the size of a single accumulation θ is described by an analogue of the equation (3):

$$\frac{d\theta}{\mathrm{d}t} = f(\theta) + g(\theta) \cdot \xi(t). \qquad (5)$$

Here t is the time; $f(\theta)$ is a deterministic function that describes the dynamics of a system; $g(\theta)$ is the amplitude of the dynamic stochastic component; $\xi(t)$ is the stochastic component.

Equation (5), under certain assumptions, corresponds to the Fokker-Planck equation for the distribution density $\varphi(\theta)$:

$$\frac{\partial\varphi(\theta)}{\mathrm{d}t} = \frac{\partial}{\partial\theta}(f(\theta)\cdot\varphi(\theta)) + \frac{1}{2}\frac{\partial^2}{\partial\theta^2}(g^2(\theta)\cdot\varphi(\theta)).$$

For the stationary case, by specifying the functions $f(\theta)$ and $g(\theta)$, we can obtain:

$$f(\theta) = a;\ g(\theta) = -b \cdot \theta;\ \varphi(\theta) = \varphi_0 \cdot e^{\frac{-2a}{b^2 \cdot \theta}} \cdot \frac{1}{\theta^2}. \qquad (6)$$

Asymptotically ($\theta \to \infty$) $\varphi(\theta)$ from (6) tends to the form (1) with an exponent of ~ 2.

Within the framework of the proposed approach, it is possible to estimate the parameters a and b for real OGS, but a number of substantive points and questions remains unclear:

1. It is easy to propose other types of functions $f(\theta)$ and $g(\theta)$, leading to a distribution of type (1);
2. It is not obvious that the stationarity condition is sufficiently satisfied for real OGS;
3. The nature and type of the stochastic component ξ raises questions;
4. It is clear that in a specific OGS the functions $f(\theta)$ and $g(\theta)$ must be individual for each accumulation and the size distribution for the entire set of accumulations must take this circumstance into account.

The list of such questions can be expanded and should be considered with further research.

3. Conclusion

The success of the application of the methodology for forecasting the most important element of the resource structure of large oil and gas systems — the distribution of hydrocarbon accumulations — created by the efforts of domestic researchers is beyond doubt. The existing approach is based on the empirically established power-law nature of this distribution, at least in the area of sufficiently large accumulations. The absence of a clear, well-founded theoretical model for the formation of hydrocarbon accumulations gives rise to a number of problems, including practical ones. First of all, this concerns the forecast of the number of small accumulations and hydrocarbon resources concentrated in them in well-studied OGS. There are grounds to assume that extrapolation of the type of distribution established by the set of identified large accumulations is not always adequate. Another problem is related to the forecast of the distribution of accumulations of hydrocarbons for poorly studied oil and gas systems. The validity of the forecast will be strengthened if the nature of the relationship between the distribution parameters and the geological characteristics of the oil and gas system is established. The explanation of some empirically established regularities, in particular the stable nature of the value of the parameter $\lambda \sim 2$, is of independent theoretical interest.

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